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Effect of a high frequency external field on transverse magnetic drift mode oscillation

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The stabilizing effect of a high frequency external field $\boldsymbol{E}_0 \cos \nu t$ on drift oscillations in an inhomogeneous plasma has been studied. It is found that for $T_e > T_i$ if $\nu \lesssim |\omega_{H_e}|$ where ω_{H_e} is the electron cyclotron frequency, non-resonant mode coupling caused by $\boldsymbol{E}_0 \cos \nu t$, diminishes the growth rate of unstable magnetic drift modes and actually stabilizes the mode if it is weakly unstable. For $\nu \gtrsim |\omega_{H_e}|$ the growth rate is accentuated.

INTRODUCTION

Resonant mode coupling in a plasma under the influence of a high frequency external field has been the subject of a number of recent investigations. If the external field frequency $\nu \approx \omega_1 - \omega_2$, where ω_1 and ω_2 are the frequencies corresponding to two stable modes of oscillation and the external field energy density exceeds some threshold value, both modes acquire a positive growth rate. A different situation may arise if the coupling is non-resonant. Let $\epsilon_1(\omega)$ and $\epsilon_2(\omega)$ be the dielectric functions (in particular the functions ϵ_1 and ϵ_2 may be identical) corresponding to two different modes of oscillations which are coupled by the external field $\boldsymbol{E}_0 \cos \nu t$. The modified dispersion relation for oscillations with frequency in the neighbourhood of $\omega = \omega_1$ is, to the order E_0^2, of the form

$$\epsilon_1(\omega) - 0(E_0^2)\left[\frac{\phi(\omega-\nu)}{\epsilon_2(\omega-\nu)} + \frac{\phi(\omega+\nu)}{\epsilon_2(\omega+\nu)}\right] = 0$$

where the function ϕ depends on the type of modes coupled. Suppose $\omega = \omega_1$ is a root of $\epsilon_1(\omega) = 0$ and $\mathrm{Im}\,\epsilon(\omega = \omega_1) > 0$, implying that $\omega = \omega_1$ is a growing oscillation. If $\omega_1 \pm \nu$ is not resonant *i.e.*, $\epsilon_2(\omega_1 \pm \nu) \neq 0$ and $\mathrm{Im}\,\epsilon_2(\omega_2 \pm \nu) < 0$ then the term $0(E_0^2)$ indicates that the external field has a stabilizing effect. In fact it has been shown (Rebhan 1969, Nunez 1971) that such non-resonant coupling increases the Landau damping of Langmuir oscillations (in this case $\epsilon_1 \equiv \epsilon_2$, $\omega \approx \omega_{pe}$ the plasma frequency, $\omega_2 = \omega_i$ the ion acoustic frequency and $\mathrm{Im}\,\epsilon_1 \equiv \mathrm{Im}\,\epsilon_2 < 0$).

In the present work we have investigatedt he possibility of such an occurrence. It is well known that in a weakly inhomogeneous plasma the transverse magnetic drift mode is unstable when $d \ln n_0/d \ln B_0 < 0$ (Krall & Rosenbluth 1963). If the external field $\boldsymbol{E}_0 \cos \nu t$ acts parallel to the magnetic field $\boldsymbol{B}_0$, transverse oscillations polarized along $\boldsymbol{B}_0$ are coupled to longitudinal oscillations perpendicular to $\boldsymbol{B}_0$. Using local approximation theory we have shown that for $\nu \ |\omega \lesssim_{H_e}|$ the above mentioned non-resonant coupling leads in general to decrease in the growth rate and under suitable conditions the instability may be actually suppressed. However, in view of the non-resonant nature of the coupling only weakly unstable modes ($ka_e \ll 1$ and $T_e/T_i \gtrsim 1$ where a_e = electron gyroradius) can be actually stabilized.

Basic Equations and Dispersion Relation

Consider an inhomogeneous plasma with a density gradient in a magnetic field $\boldsymbol{B}_0 = \hat{z} B_0(x)$ and acted upon by a high frequency external field $\boldsymbol{E}_0 = \hat{z} E_0 \cos \nu t$.

We start from the first order linearized Vlasov equation for charge of species α;

$$\frac{\partial f_1^{(\alpha)}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f_1^{(\alpha)}}{\partial \boldsymbol{r}} + \frac{e_\alpha}{m_\alpha} \left(\boldsymbol{E}_0 \cos \nu t + \frac{\boldsymbol{v} \times \boldsymbol{B}_0}{c} \right) \cdot \frac{\partial f_1^{(\alpha)}}{\partial \boldsymbol{v}}$$

$$= -\frac{e_\alpha}{m_\alpha} \left[\boldsymbol{E}(\boldsymbol{r}, t) + \frac{\boldsymbol{v} \times \boldsymbol{B}}{c} \right] \frac{\partial f_0^{(\alpha)}}{\partial \boldsymbol{v}} . \qquad \dots \quad (1)$$

$f_0^{(\alpha)}$ the equilibrium distribution function is chosen to be nearly Maxwellian,

$$f_0^{(\alpha)} = (2\pi v_\alpha^2)^{-3/2} n_0(x + v_y/\omega_{H\alpha}) \exp\left[-\frac{1}{2v_\alpha^2} \left\{ v_\perp{}^2 + \left(v_z - \frac{e_\alpha E_0}{m_\alpha \nu} \sin \nu t \right)^2 \right\} \right]$$

$$\omega_{H\alpha} = \frac{e_\alpha B_0}{m_\alpha c}; \quad \omega_{p\alpha}^2 = 4\pi n_0 e_\alpha^2/m_\alpha \qquad \dots \quad (2)$$

and

$$\boldsymbol{E}_0 = \hat{z} E_0 = \hat{z}(\mathrm{E}_0 + \mathrm{E}_I) = \hat{z} \frac{\nu^2}{\nu^2 - \omega_{pe}^2 - \omega_{pi}^2} \mathrm{E}_0 \qquad \dots \quad (3)$$

where E_I is the internal field arising in the quiscent plasma in response to the external field E (Robhan 1969).

Consider perturbations propagating in the y-direction (transverse to the external magnetic field $\boldsymbol{B}_0$)

$$\boldsymbol{E} = \sum_{n=-\infty}^{+\infty} \boldsymbol{E}^{(n)}(k, \omega) \exp\left(iky - i(w+\nu)t\right) \qquad \dots \quad (4)$$

In a low β plasma (β = plasma pr/magnetic pr) one can neglect the $E_x^{(n=0)}$ component (perpendicular to $\boldsymbol{B}_0$) for perturbations of the form (4) for drift modes (Krall & Rosenbluth 1963). For high frequency oscillations longitudinal and transverse components propagating perpendicular to magnetic field $\boldsymbol{B}_0$ are coupled. This coupling can however be neglected for $1 \gg \omega_{pe}^2/c^2k^2 = \frac{1}{2}\beta \frac{\omega_{He}^2}{k^2v_e^2}/(1+T_e/T_i)$ where $T_e(T_i)$ is the electronic (ionic) temperature. Thus for $\beta < \frac{m_e}{m_i}$ we are justified neglecting $E_x^{(n)}(k, \omega)$ $(n = \pm 1, \pm 2, \dots)$ for oscillations with wave lengths

$$1/k \ll \left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} \frac{v_e}{|\omega_{He}|} (1+T_i/T_e)^{\frac{1}{2}} \qquad \dots \quad (5)$$

$$= a_e[(1+T_i/T_e)m_i/m_e]^{\frac{1}{2}}$$

We shall assume (5) to be valid. Only two components of the perturbed electric field need therefore be considered : the longitudinal component $E_y^{(n)}$ and the transverse one $E_z^{(n)}$ which are coupled by the external field $\boldsymbol{E}_0 \cos \nu t$

$$\boldsymbol{E} = \sum_{n=-\infty}^{+\infty} [\hat{y}E_y^{(n)} + \hat{z}E_z^{(n)}) \exp(iky - i(\omega+n\nu)t). \qquad \dots \quad (6)$$

Transforming $\boldsymbol{v}$ to $\boldsymbol{w}$

$$\boldsymbol{w} = \boldsymbol{v} - \hat{z}\,\frac{e_\alpha E_0}{m_\alpha \nu} \sin \nu t \qquad (7)$$

equation (1) is now solved by applying the usual method of integrating along unperturbed orbits given by (Krall 1968).

$$w_z' \equiv w_z'(t') = w_z$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(x' + w_y'/\omega_{H\alpha}) = \frac{\mathrm{d}}{\mathrm{d}t}(w_\perp'^2) = 0$$

$$w_y' \equiv w_y'(t') = w_\perp \sin(\theta - \omega_{H\alpha}\overline{t'-t}) + w_D$$

$$y' \equiv y'(t') = y + \frac{w_\perp}{\omega_{H\alpha}} \cos(\theta - \omega_{H\alpha}\overline{t'-t}) - \frac{w_\perp}{\omega_{H\alpha}} \cos\theta + w_D(t'-t)$$

$$w_D = \frac{1}{2}\frac{w_\perp^2}{\omega_{H\alpha}}\frac{1}{B_0}\frac{dB_0}{dx} = \tfrac{1}{2}\epsilon w_\perp^2/\omega_{H\alpha}$$

$$\epsilon = \frac{1}{B_0}\frac{dB_0}{dx} \qquad \dots \quad (8)$$

From (1), (6), (7) and (8) we get

$$f_1^{(\alpha)} = -\frac{ie_\alpha}{m_\alpha}\sum_{n=-\infty}^{+\infty}\sum_{l,m=-\infty}^{+\infty}(i)^{l-m}\frac{J_l J_m \exp(iky-i\overline{\omega+n\nu}\,t)}{D(n,l)}\exp i(l-m)\theta$$

$$\times\Big[E_z^{(n)}\Big(-\frac{w_z}{v_\alpha^2}f_0^{(\alpha)}+\frac{kw_z}{\omega+n\nu}\frac{1}{\omega_{H\alpha}}\frac{\partial f_0^{(n)}}{\partial x}\Big)+\Big\{E_y^{(n)}+\frac{\mu_\alpha}{2i}\frac{\nu}{\omega+n\nu}\Big(\frac{D(n,l)}{D(n-1,l)}e^{i\nu t}$$

$$-\frac{D(n,l)}{D(n+1,l)}e^{-i\nu t}\Big\}\Big(\frac{l\omega_{H\alpha}}{kv_\alpha^2}f_0^{(\alpha)}+\frac{1}{\omega_{H\alpha}}\frac{\partial f_0^{(\alpha)}}{\partial x}\Big)\Big] \qquad \dots (9)$$

where

$$\mu_\alpha = ke_\alpha E_0/m_\alpha\nu^2 \qquad \dots (10)$$

$$D(n,l) = \omega+n\nu+l\omega_{H\alpha}-\epsilon k w_\perp^2/\omega_{H\alpha} \qquad \dots (11)$$

$$J_l \equiv J_l(kw\ /\omega_{H\alpha})$$

and θ is the azimuthal angle variable in velocity space.

The first order charge density ρ and current density j_z are given by

$$\rho = \sum_\alpha e_\alpha \int f_1^{(\alpha)}\mathrm{d}\boldsymbol{v} = \sum_\alpha e_\alpha\int \delta_1^{(\alpha)}(\boldsymbol{w},\boldsymbol{n},t)\mathrm{d}\boldsymbol{w}$$

$$= \sum_\alpha\sum_{n=-\infty}^{+\infty}\rho_{II}^{(\alpha)(n)}(k,\omega)\exp(iky-i(\omega+n\nu)t) \qquad \dots (12a)$$

where

$$\rho^{(\alpha)(n)}(k,\omega) = i\frac{n_0 e_\alpha^2}{m_\alpha}\sum_{l=-\infty}^{+\infty}\int \mathrm{d}w_\perp\frac{w_\perp}{v_\alpha^2}\exp(-w_\perp^2/2v_\alpha^2)\frac{J_l^2(kw_\perp/\omega_{H\alpha})}{D(n,l)}$$

$$\times\Big(-\frac{l\omega_{H\alpha}}{kv_\alpha^2}-\frac{l\omega_{H\alpha}\epsilon'}{k^2v_\alpha^2}+\frac{\epsilon'}{\omega_{H\alpha}}\Big)\Big[E_y^{(n)}+\frac{\mu_\alpha}{2i}\Big(\frac{\nu}{\omega+\overline{n+1}\nu}E_z^{(n-1)}-\frac{\nu}{\omega+\overline{n-1}\,\nu}E_z^{(n-1)}.\Big)\Big] \qquad \dots (12b)$$

$$j_z = \sum_\alpha\int vf_1^{(\alpha)}\mathrm{d}\boldsymbol{v} = \sum_\alpha\int w_z f_1^{(\alpha)}(\boldsymbol{w},\boldsymbol{r},t)d\boldsymbol{w}+\frac{e_\alpha E_0}{m_\alpha\nu}\sin\nu t\int f_1^{\alpha}(\boldsymbol{w},\boldsymbol{r},t)\mathrm{d}\boldsymbol{w}$$

$$= \sum_\alpha\sum_{n=-\infty}^{+\infty}\exp(iky-i(\omega+n\nu)t)j_z^{(\alpha)(n)} \qquad \dots (13a)$$

$$j_z^{(\alpha)(n)} = E_z^{(n)}\Big(i\frac{n_0e_\alpha^2}{m_\alpha}\Big)\sum_{l=-\infty}^{+\infty}\int \mathrm{d}w_\perp\frac{w_\perp}{v_\alpha^2}\exp(-w_\perp^2/2v_\alpha^2)\frac{J_l^2(kw_\perp/\omega_{H\alpha})}{D(n,l)}\times$$

$$\times\Big(-1-\frac{l}{k}\epsilon'+\frac{kv_\alpha^2}{\omega+n\nu}\frac{1}{\omega_{H\alpha}}c'\Big)+\frac{1}{2i}\frac{e_\alpha E_0}{m_\alpha\nu}(\rho^{(\alpha)(n+1)}-\rho^{(\alpha)(n-1)}) \qquad \dots (13b)$$

$$\epsilon' = \frac{1}{n_0}\frac{\mathrm{d}n_0}{\mathrm{d}x} \qquad \dots (14)$$

Substituting these expressions for ρ and j_z in the field equations

$$ikE_y{}^{(n)} = 4\pi \sum_{\alpha} \rho^{(\alpha)(n)} \qquad \ldots \quad (15)$$

$$[c^2k^2-(\omega+n\nu)^2]E_z^{(n)}-4\pi i(\omega+n\nu)\sum_{\alpha} j_z^{(\alpha)(n)} = 0 \qquad \ldots \quad (16)$$

we obtain the system of coupled equations

$$\epsilon_L(\omega+n\nu)E_y^{(n)}+\frac{i\mu_e}{2}\,\chi_e(\omega+n\nu)\frac{\nu}{\omega+\overline{n+1}\,\nu}\,E_z^{(n+1)}-\frac{i\mu_e}{2}\,\chi_e(\omega+n\nu)\frac{\nu}{\omega+\overline{n-1}\,\nu}$$

$$\times E_z^{(n-1)} = 0 \qquad (17a)$$

$$[\epsilon_T(\omega+n\nu)+\tfrac{1}{4}\mu_e{}^2\nu^2\chi_e(\omega+n-1\nu)+\tfrac{1}{4}\mu_e{}^2\nu^2\chi_e(\omega+n+1\nu)]E_z{}^{(n)}$$

$$+\frac{i\mu_e}{2}\,\nu(\omega+n\nu)[\chi_e(\omega+\overline{n+1}\,\nu)E_y^{(n+1)}-\chi_e(\omega+\overline{n-1}\,\nu)E_y^{(n-1)}] = 0 \qquad \ldots \quad (17b)$$

where

$$\epsilon_L(\omega+n\nu) = 1+\sum_{\alpha}\chi^{(\alpha)}(\omega+n\nu) \qquad \ldots \quad (18)$$

$$\chi^{(\alpha)}(\omega+n\nu) = \frac{\omega_{p\alpha}^2}{k^2v_\alpha^2}\sum_{l=-\infty}^{+\infty}\int dw_\perp \frac{w_\perp}{v_\alpha^2}\exp(-w_\perp{}^2/2v_\alpha^2)\frac{J_l^2\left(\dfrac{kw_\perp}{\omega_{H\alpha}}\right)}{\omega+n\nu+l\omega_{H\alpha}-\dfrac{ckw_\perp{}^2}{2\omega_{H\alpha}}}$$

$$\times\left(l\omega_{H\alpha}-\frac{l^2\omega_{H\alpha}\epsilon'}{k}+\frac{kv_\alpha^2}{\omega_{H\alpha}}\,\epsilon'\right) \qquad W \quad (19)$$

$$\epsilon_T(\omega+n\nu) = c^2k^2-(\omega+n\nu)^2-(\omega+n\nu)\sum_{\alpha}\omega_{p\alpha}^2\sum_{l=-\infty}^{+\infty}\int dw_\perp\frac{w_\perp}{v_\alpha^2}$$

$$\times\exp(-w^2/2v_\alpha^2)\frac{J_l{}^2\left(\dfrac{kw_\perp}{\omega_{H\alpha}}\right)}{\omega+n\nu+l\omega_{H\alpha}-\dfrac{\epsilon kw_\perp{}^2}{2\omega_{H\alpha}}}\left(-1-\frac{l}{k}\,\epsilon'+\frac{kv_\alpha^2\epsilon'}{\omega_{H\alpha}\omega+n\nu}\right) \quad \ldots \quad (20)$$

The modulation effect of the external field on ion motion has been neglected. Eliminating $E_y^{(\pm 1)}$ from (17b) with the help of (17a) we now obtain the dispersion relation for the transverse mode $E_z^{(0)}$ correct to second order in the small parameter μ_e

$$\epsilon_T(\omega)+\frac{1}{4}\,\mu_e^2\nu^2-\frac{1}{4}\,\mu_e^2\nu^2\left\{\frac{[1+\chi_e(\omega-\nu)]^2}{\epsilon_L(\omega-\nu)}+\frac{[1+\chi_e(\omega+\nu)]^2}{\epsilon_L(\omega+\nu)}\right\} = 0 \qquad \ldots \quad (21)$$

For $B_0 = 0$, $\epsilon' = 0$, *i.e.*, for a homogeneous plasma without magnetic field, (21) is identical with the dispersion relation correct to the order of $E_0{}^2$ given previously (Prasad 1968, Montgomery & Alexeff 1966)

Roots of the Dispersion Relation and Discussions

In absence of the external field the dispersion relation is given by $\epsilon_T(\omega) = 0$. It is known that for

$$\epsilon/\epsilon' = \frac{\mathrm{d}}{\mathrm{d}x}\ln B_0 / \frac{\mathrm{d}}{\mathrm{d}x}\ln n_0 < 0 \qquad \ldots \quad (22)$$

the transverse magnetic drift mode is unstable. In this case (Krall & Rosenbluth 1963) the frequency and growth rate are given by $\omega = \omega_0 + i\gamma_0$ with

$$\omega_0 = -\frac{k\epsilon' v_e^2}{|\omega_{He}|}\,(1+c^2k^2/\omega_{pe}^2)^{-1} \qquad \ldots \quad (23)$$

$$\gamma_0 = -\pi(\epsilon'/\epsilon)\,|\omega_0|\;\frac{m_e}{m_i}\;\frac{1+c^2k^2/\omega_{pe}^2+T_e/T_i}{(1+c^2k^2/\omega_{pe}^2)^2}$$

$$\times J_0^2(ka_i\sqrt{\alpha})e^{-\alpha} \qquad \ldots \quad (24)$$

$$\alpha = \epsilon'/\epsilon \cdot \frac{T_e/T_i}{1+c^2k^2/\omega_{pe}^2}; \quad a_\alpha = v_\alpha/|\omega_{H\alpha}|$$

The value of ϵ/ϵ' is obtained from the zero order (equilibrium) field equations

$$(\nabla\times\boldsymbol{B}_0)_y = \frac{4\pi}{c}(j_0)$$

$$= \frac{4\pi}{c}\,\Sigma\, e \int v_y f_0^{(\alpha)}(x, \boldsymbol{v})\mathrm{d}\boldsymbol{v}$$

Substituting from (2) and neglecting $\partial^2 n_0/\partial x^2$ and higher order derivatives we get:

$$-\frac{\epsilon}{\epsilon'} = \frac{1}{2}\cdot\frac{\sum_\alpha \frac{1}{2}n_0 m\, v_\alpha^2}{\frac{1}{8\pi}B_0^2} = \tfrac{1}{2}\beta \qquad \ldots \quad (25)$$

Substituting for $|\epsilon'/\epsilon|$ and using the relation

$$\frac{c^2k^2}{\omega_{pe}^2} = \frac{2}{\beta}\left(1+\frac{T_i}{T_e}\right)k^2a_e^2 \qquad \ldots \quad (26)$$

we find that for $ka_e \ll 1$ the growth rate given by (24) is low except when $T_e/T_i \ll 1$.

We shall now discuss the roots of (21) corresponding to different possibilities :

Case I : $\epsilon/\epsilon' < 0$; $\epsilon > 0$, $\epsilon' < 0$

From the expression for $\epsilon_L(\omega \pm \nu)$ as given by (18) and (19) one easily finds that for $\epsilon > 0$ the electron velocity integral $\int \mathrm{d}w_\perp w_\perp \frac{\exp(-w_\perp^2/2v_\alpha^2)J_l^2}{D(n, l)}$ is resonant

$(\omega_{H_e} = -|\omega_{H_e}|)$ provided $\omega \pm \nu + l\omega_{H_e} < 0$ while the ion velocity integral is resonant for $\omega \pm \nu + l\omega_{H_i} > 0$. Splitting $\epsilon_L(\omega \pm \nu)$ into real and imaginary parts

$$\epsilon_L'(\omega \pm \nu) = \mathrm{Re}\,\epsilon_L(\omega \pm \nu) = 1 + \mathrm{Re}\chi_e(\omega \pm \nu) + \mathrm{Re}\chi_i(\omega \pm \nu).$$

$$= 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{k^2 v_\alpha^2} \sum_l l\omega_{H\alpha} P \int_0^\infty \frac{dw_\perp w_\perp}{v_\alpha^2} \exp(-w_\perp^2/2v_\alpha^2) \frac{J_l^2\left(\dfrac{kw_\perp}{\omega_{H\alpha}}\right)}{\omega \pm \nu + l\omega_{H\alpha} - \dfrac{k\epsilon}{2\omega_{H\alpha}} \omega_\perp^2} \quad \ldots \quad (27)$$

$$\epsilon_L''(\omega \pm \nu) = \mathrm{Im}\,\epsilon_L(\omega \pm \nu) = \mathrm{Im}\,\chi_e(\omega \pm \nu) + \mathrm{Im}\,\chi_i(\omega \pm \nu)$$

$$= -\pi|\epsilon'/\epsilon| \frac{\omega_{pe}^2}{k^2 v_e^2} (1 + c^2k^2/\omega_{pe}^2)^{-1} \sum_{l=-\infty}^{+\infty} \frac{l\omega_{H_e}}{\omega_0}$$

$$\times \left[\theta(-\omega \mp \nu - l\omega_{He}) J_l^2\left(\sqrt{2}ka_e \sqrt{\left|\frac{\epsilon'}{\epsilon}\right| \frac{\omega + \nu \pm l\omega_{He}}{\omega_0(1 + c^2k^2/\omega_{pe}^2)}} \right) \exp\left\{ -\left|\frac{\epsilon'}{\epsilon}\right| \left|\frac{\omega \pm \nu + l\omega_{He}}{\omega_0(1 + c^2k^2/\omega_{pe}^2)}\right| \right\} \right.$$

$$+ \theta(\omega \pm \nu + l\omega_{Hi}) \frac{\omega_{pi}^2}{\omega_{pe}^2} \left(\frac{T_e}{T_i}\right)^2 J_l^2 \left(\sqrt{2}ka_i \sqrt{\left|\frac{\epsilon'}{\epsilon}\right| \frac{\omega \pm \nu + l\omega_{Hi}}{\omega_0(1 + c^2k^2/\omega_{pe}^2)}} \right)$$

$$\left. \exp\left\{ -\left|\frac{\epsilon'}{\epsilon}\right| \frac{\omega \pm \nu + l\omega_{Hi}}{\omega_0(1 + c^2k^2/\omega_{pe}^2)} \right\} \right] \quad \ldots \quad (28)$$

where $\theta(x) = 1$ or 0 according as $x > 0$ or $x < 0$. It is evident that for $\omega \pm \nu + l\omega_{He} < 0$, the ion terms are small compared to the electron terms. If we restrict the external field frequency ν to values close to $|\omega_{He}|$ the major contribution comes from $l = \pm 1$ terms. One easily finds that under the condition

$$\frac{|\omega_{He}|}{\nu} > 1 + \frac{\omega}{\nu} \gtrsim 1 \quad \ldots \quad (29)$$

the $l = \pm 1$ term is important and we have

$$\epsilon_L''(\omega + \nu) = \pi|\epsilon'/\epsilon| \left|\frac{\omega_{He}}{\omega_0}\right| \frac{\omega_{pe}^2}{k^2 v_e^2} (1 + c^2k^2/\omega_{pe}^2)^{-1} J_1^2(\sqrt{2}ka_e\sqrt{\Delta})e^{-\Delta}$$

$$\epsilon_L''(\omega - \nu) \approx 0 \quad \ldots \quad (30)$$

with

$$\Delta = \left|\frac{\epsilon'}{\epsilon}\right| \left|\frac{\omega + \nu + \omega_{He}}{\omega_0}\right| (1 + c^2k^2/\omega_{pe}^2)^{-1}$$

$$= (ka_e)^{-2} x/(1 + T_i/T_e) \quad \ldots \quad (31)$$

where we have substituted

$$x = \left|\frac{\omega + \nu + \omega_{He}}{\omega_0}\right| = \frac{|\omega_{He}| - \nu - \omega}{\omega_0}. \quad \ldots \quad (32)$$

and used (25) and (26).

Similarly for $$1 \approx 1-\frac{\omega}{\nu} > \frac{\omega_{He}}{\nu} \quad \ldots (33)$$

the term with $l = -1$ is important and we have

$$\epsilon_L''(\omega+\nu) \approx 0$$

$$\epsilon_L''(\omega-\nu) \simeq -\pi\left|\frac{\epsilon'}{\epsilon}\right|\left|\frac{\omega_{He}}{\omega_0}\right|\frac{\omega_{pe}^2}{k^2v_e^2}(1+c^2k^2/\omega_{pe}^2)^{-1}J_1^2(\sqrt{2}ka_e\sqrt{\Delta'})e^{-\Delta'} \quad \ldots (34)$$

with

$$\Delta' = \left|\frac{\epsilon'}{\epsilon}\right|\left|\frac{\omega-\nu-\omega_{He}}{\omega_0}\right|(1+c^2k^2/\omega_{pe}^2)^{-1}$$

$$= (ka_e)^{-2}x'/(1+T_i/T_e) \quad \ldots (35)$$

$$x' = \frac{\nu-\omega-|\omega_{He}|}{\omega_0} \quad \ldots (36)$$

When (29) holds, the major contribution to $\epsilon_L'(\omega+\nu)$ comes from the $l = +1$ term. We have from (27)

$$\epsilon_L'(\omega+\nu) = 1+\mathrm{Re}\,\chi_e+\mathrm{Re}\,\chi_i$$

$$\simeq \frac{\omega_{pe}^2}{k^2v_e^2}\left|\frac{\omega_{He}}{\omega+\nu+\omega_{He}}\right|I_1(k^2a_e^2)e^{-k^2a_e^2} \quad \ldots (37)$$

$$\approx \frac{a_e^2}{\lambda_e^2}\left|\frac{\omega_{He}}{\omega_0}\right|\cdot\frac{1}{x}\text{ for } ka_e \gg 1 \quad \ldots (38)$$

From (30) and (37) we find for $k\alpha_e \ll 1$

$$\left|\frac{\epsilon_L''(\omega+\nu)}{\epsilon_L'(\omega+\nu}\right| \approx \pi\Delta^2 e^{k^2a_e^2-\Delta} \lesssim 0(1) \quad \ldots (39)$$

We shall now determine approximate root of (21). Putting

$$\omega = \Omega+i\Gamma \quad \ldots (40)$$

and using condition (29), (34) and (37) we get

$$\Omega = \frac{k\epsilon'v_e^2}{\omega_{He}}(1+c^2k^2/\omega_{pe}^2)^{-1}\left\{1-\frac{1}{4}\left(\frac{eE_0}{m_ec\nu}\right)^2\left(1-\frac{1}{|\epsilon_L'|^2}\right)\right\} \quad \ldots (41)$$

$$\Gamma = \gamma_0+\delta\gamma_0$$

$$= \gamma_0-\frac{\pi A}{4}\mu_e^2\left|\frac{\epsilon'}{\epsilon}\right|\frac{|\omega_{He}|}{(1+c^2k^2/\omega_{pe}^2)^2}\cdot\frac{\nu^2}{k^2v_e^2}\frac{J_1^2(\sqrt{2}ka_e\sqrt{\Delta})}{|\epsilon_L'(\omega+\nu)|^2}e^{-\Delta} \quad \ldots (42)$$

where $$A = (1+\mathrm{Re}\,\chi_i)^2/(1+\epsilon_L''/\epsilon_L'|^2) = 0(1) \quad \ldots (43)$$

Equation (41) gives the shift in the frequency of the transverse magnetic drift mode oscillation; the shift being small we can replace ω by $\Omega \approx \omega_0$ in the expression fo x. As $\delta\gamma_0$ is negative the growth rate is in general diminished whenever (29) is valid. Under favourable conditions the instability, if sufficiently weak, is actually suppressed. Consider wave lengths such that

$$ka_i \approx 1 \text{ and } ka_e = \left(\frac{m_e}{m_i}\frac{T_e}{T_i}\right)^{\frac{1}{2}} ka_i \ll 1 \qquad \ldots \quad (44)$$

hold. We may then use (38) and also take

$$J_0^2(\sqrt{2}ka_i\sqrt{\alpha}) \approx 1 \qquad \ldots \quad (45)$$

Substituting in (42) from equations (24), (25), (26) (38), (44) ad (45) we find that

$$\Gamma < 0$$

i.e., the mode is stabilized if

$$1 \gg \mu_e^2 \gg \frac{8}{A}\left(\frac{m_e}{m_i}\right)^2 (ka_i)^2 \left(\frac{\omega_{pe}^2}{\nu\omega_{He}}\right)^2 \left|\frac{\omega_{He}}{\omega_0}\right| \frac{1+T_e/T_i}{x^3} \exp\left[-\frac{1}{k^2a_e^2}\left(\frac{T_e}{T_i}-x\right)\Big/\left(1+\frac{T_i}{T_e}\right)\right] \qquad \ldots \quad (46)$$

since our calculations are valid only for $\mu_e^2 \ll 1$. For $\omega_{He} \gtrsim \nu^2 \gtrsim \omega_{pe}^2$ the right hand side of (46) $<<1$ provided

$$\frac{T_e}{T_i} > x = \frac{|\omega_{He}|-\omega_0-\nu}{\omega_0} \approx 1 \qquad \ldots \quad (47)$$

Thus the non-resonant parametric coupling between the transverse and longitudinal modes caused by the external high frequency field is able to stabilize a weak drift instability corresponding to $ka_e \ll 1$ and $T_e/T_i \gtrsim 1$.

When (33) holds contribution to the imaginary part of the parametric coupling term in (21) comes from $\epsilon_L''(\omega-\nu)$. From a comparison of (30) and (34) we find that $\epsilon_L''(\omega\pm\nu)$ have opposite signs; consequently $\delta\gamma_0$ will be positive. Thus for $\nu > |\omega_{He}|$ the external field tends to accentuate the growth rate.

Case II : $\epsilon/\epsilon' < 0$; $\epsilon < 0$, $\epsilon' > 0$

As in the previous case, ω_0 and γ_0 are given by the equations (23) and (24); thus ω_0 is negative. Consequently in this case when (29) holds it is the electron velocity integral

$$\int dw_\perp \exp(-w_\perp^2/2v_e^2)J_l^2\Big/\left[\omega-\nu+l\omega_{He}-\frac{k\epsilon w_\perp^2}{2\omega_{He}}\right)$$

occurring in the expression for $\epsilon_L(\omega-\nu)$ which is resonant when $l = -1$

Thus for

$$\left|\frac{\omega_{He}}{\nu}\right| \neq 1+\frac{|\omega|}{\nu} \approx 1 \qquad \ldots \quad (48)$$

we have

$$\epsilon_L''(\omega+\nu) \approx 0$$

$$\epsilon_L''(\omega-\nu) = +\pi \left|\frac{\epsilon'}{\epsilon}\right| \frac{\omega_{pe}^2}{k^2 v_e^2} \left|\frac{\omega_{He}}{\omega_0}\right| (1+c^2k^2/\omega_{pe}^2)^{-1} J_1^2(\sqrt{2}ka_e\sqrt{\Delta})e^{-\Delta} \quad \dots \quad (49)$$

with

$$\Delta = \left|\frac{\epsilon'}{\epsilon}\right| \left|\frac{\omega-\nu-\omega_{He}}{\omega_0}\right| (1+c^2k^2/\omega_{pe}^2)^{-1} \quad \dots \quad (50)$$

As $\omega \approx \omega_0$ is negative, Δ has the same value as in (31). Replacing (30) by (49) we find that in this case also the real and imaginary parts of the root of the dispersion equation are given by (41) and (42) respectvely, and hence the conclu sions are the same as in Case I.

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